The Role of the Marketplace Operator in Inducing Competition

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Joint work with

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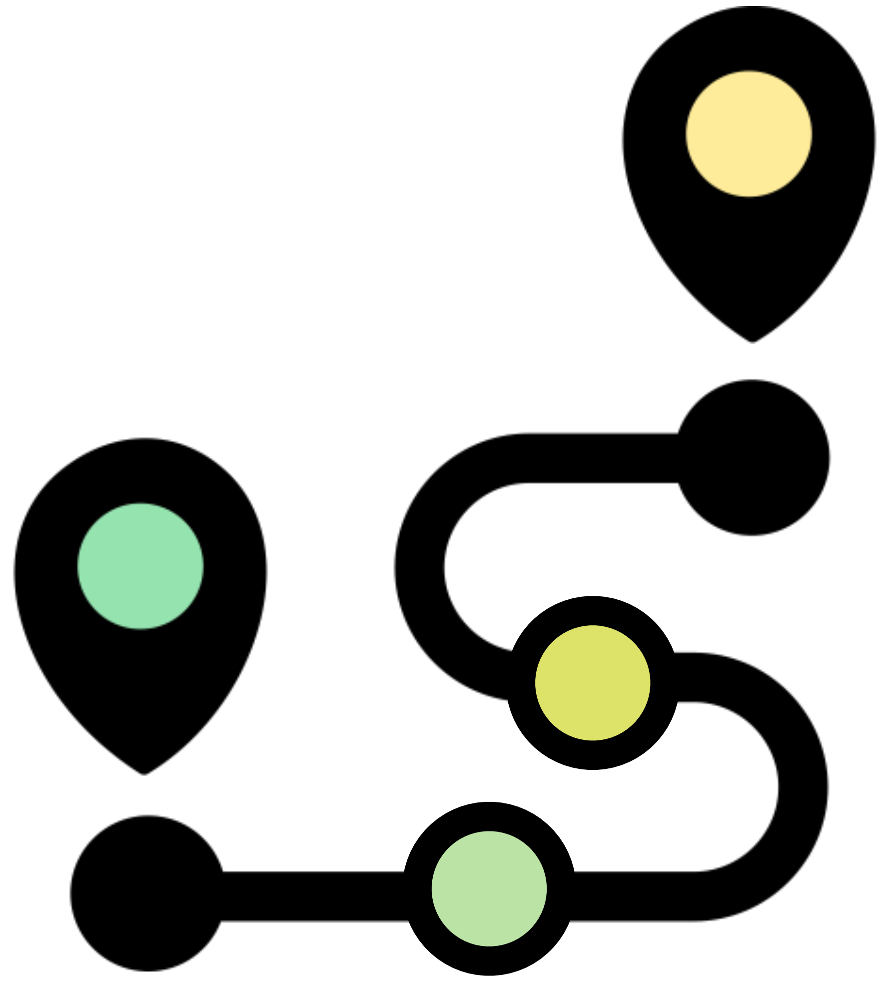
Talk roadmap

1. **Introduce the problem setting** and (hopefully) convince you that it is a relevant and interesting problem to study
2. **Present our model** for this problem
3. **Explain our main results** + high level idea of how these results are obtained (no complicated complex mathematical tools needed, just a matter of constructing arguments by connecting basic ideas)
4. **Highlight some practical implications** of our main results



Motivation

Suppose you operate a farmer's market.

In return for taking care of logistics, the farmers pay you 5% of their revenue.

One day, Farmer Joe realizes he is the only farmer selling carrots and they are in high demand, so he increases his price by 200%.

This makes the market-goers very unhappy and they begin going to other farmer's markets instead of yours.

You can't control how Farmer Joe sets his prices, but you want to somehow induce him to lower his price, in order to keep market-goers happy.

Idea: you can enter the market as a competing carrot seller!

How can you induce Farmer Joe to compete?

Some considerations:

1. ***You shouldn't set a price that is too high*** (i.e., higher than the price Farmer Joe would otherwise set).
2. ***You shouldn't set a price that is too low*** (i.e., so low that Farmer Joe cannot make a profit if he matches your price).
3. ***You need to be a "credible seller."*** If you set a competitive price but only have 10 carrots for sale, Farmer Joe will just wait for you to sell out and charge a higher price.

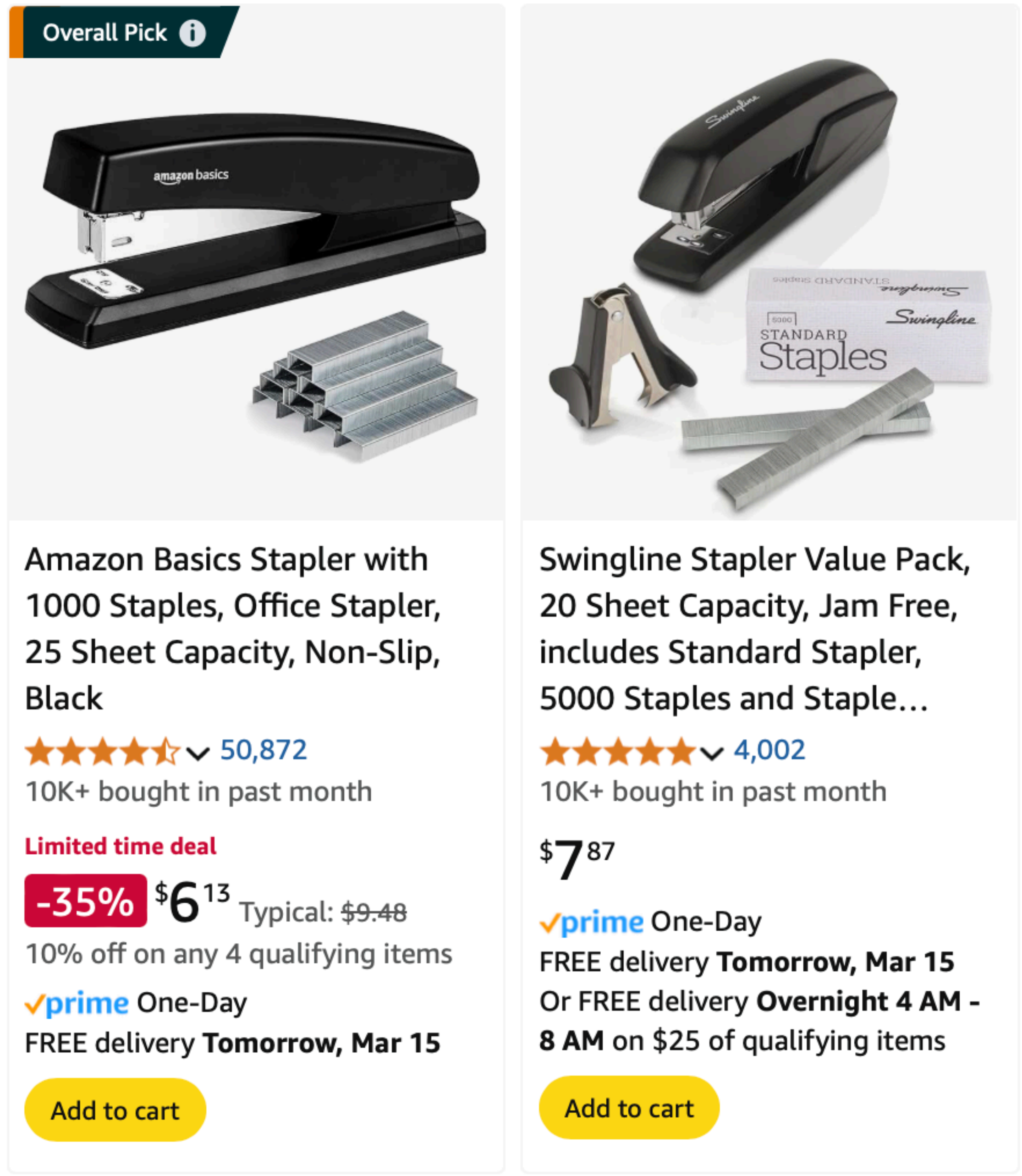
Inducing competition is a delicate task

Guiding questions

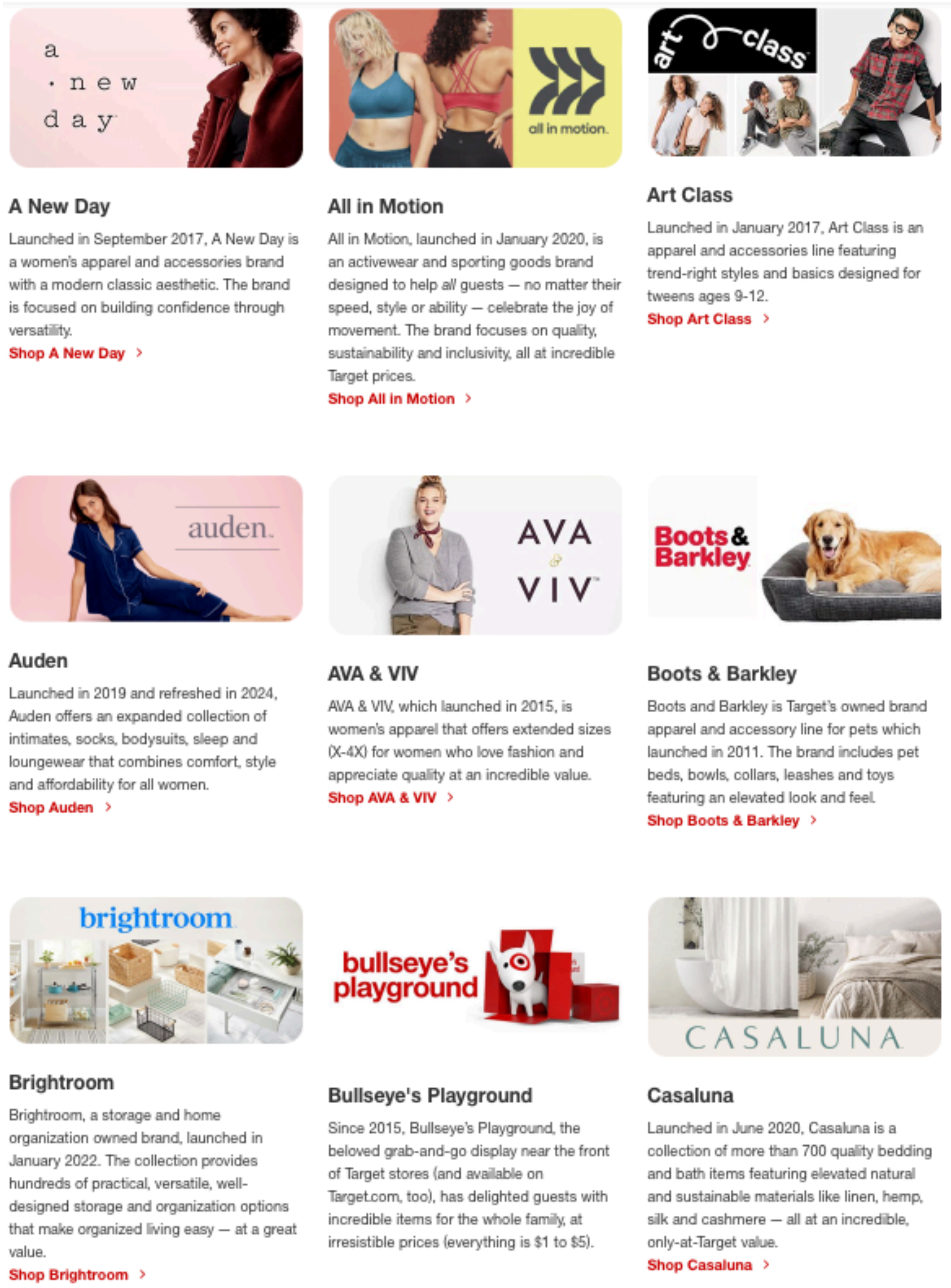
1. How can the marketplace operator set their price and inventory to induce competition?
2. When is it beneficial for the marketplace operator to induce competition?
3. What are the implications for consumer surplus and total welfare?

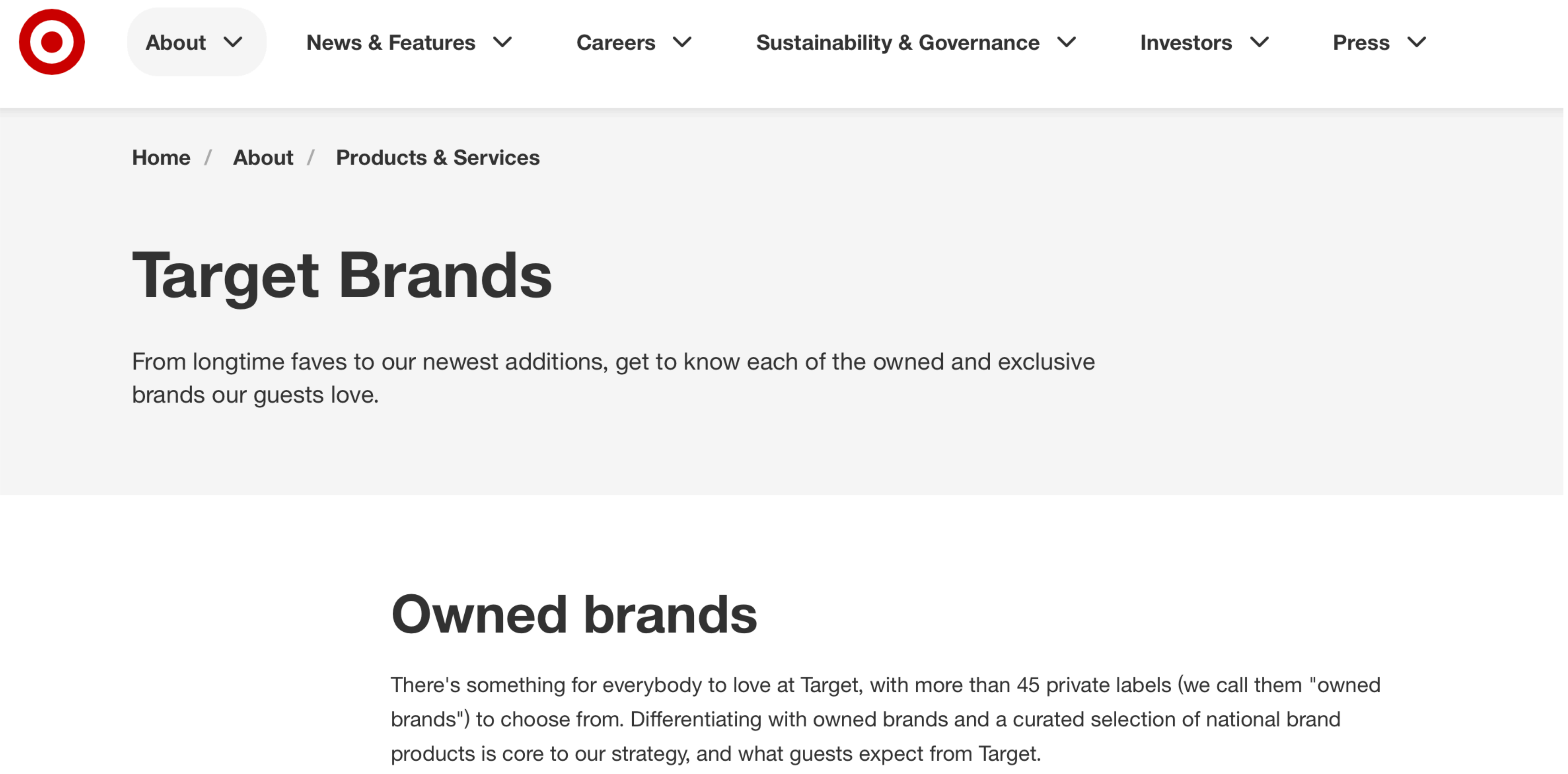
Some real examples of marketplace operators who are also sellers

Amazon



Target





Walmart

Background: Classical duopoly models

“Standard” duopolies (both firms are profit-maximizing)

pre-1900s: Work on duopolies with simultaneous actions and a single decision variable (*either* price or quantity)

- In a Cournot duopoly (Cournot, 1897), sellers A and B simultaneously choose their quantities q_A and q_B, which determines the price $p = f(q_A + q_B)$
- In a Bertrand duopoly (Bertrand, 1883), sellers simultaneously choose their prices p_A and p_B, then the lower-priced seller $i \in \{A, B\}$ gets demand $D(p_i)$ and the other gets zero demand. When $p_A = p_B$ each seller gets demand $D(p_i)/2$

1934: Stackelberg analyses a duopoly with sequential actions and a single decision variable (quantity)

mid-late 1900s: Work on duopolies with two decision variables (price *and* quantity) in both simultaneous and sequential settings (Shubik, 1959; Levitan and Shubik, 1978; Kreps and Scheinkmen, 1983; Davidson and Deneckere, 1986; Boyer and Moreaux 1987, 1989)

Mixed oligopolies

In a mixed oligopoly, there is a

welfare-maximizing public firm
(e.g., the government)

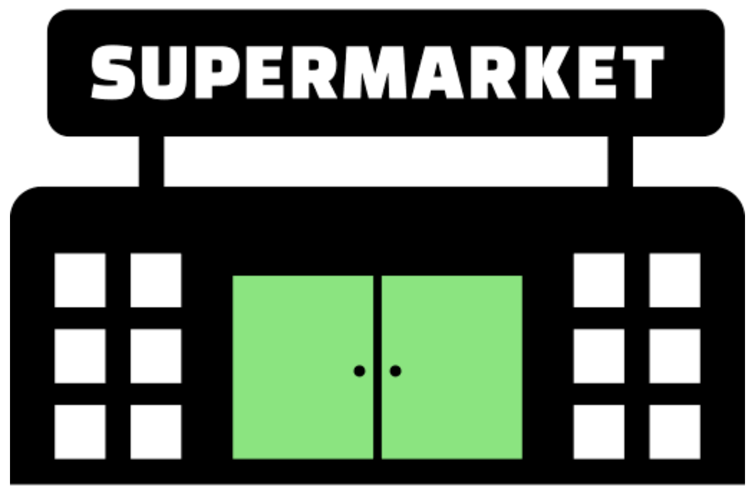
and

a profit-maximizing private firm

(Cremer et al., 1989; De Fraja and Delbono, 1990)

We will consider a sequential duopoly with two decision variables where one firm is profit-maximizing and the other firm is “profit+welfare”-maximizing

Model



Marketplace Operator (MO)

Big, has to make decisions far in advance

Cares about profit but also customer satisfaction

Independent Seller (IS)

Smaller, can make decisions more reactively

Solely cares about maximizing their own profit

Pays commission/referral fee to MO

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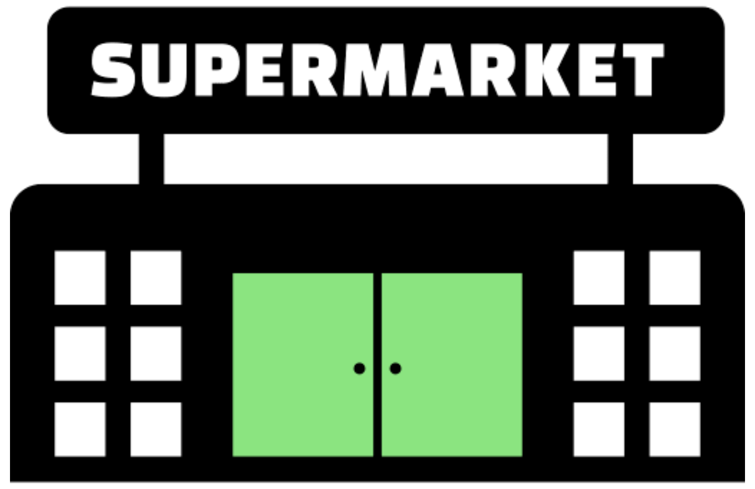
$\rightarrow$ Stackelberg duopoly where MO = leader and IS = follower

Stage 1: MO chooses their price $p_{\text{MO}} \geq 0$ and quantity $q_{\text{MO}} \geq 0$.

Stage 2: IS observes $p_{\text{MO}}, q_{\text{MO}}$ and chooses their price $p_{\text{IS}} \geq 0$ and quantity $q_{\text{IS}} \geq 0$.



Marketplace Operator (MO)

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Cares about profit but also customer satisfaction

Solely cares about maximizing their own profit

Pays commission/referral fee to MO

α = referral fee paid by IS to MO
k = MO's additional benefit per sale (due to customer satisfaction that contributed to marketplace health)
c_{MO} = MO's per-unit cost
c_{IS} = IS's per-unit cost

$$u_{\mathrm{MO}} = (p_{\mathrm{MO}} + k)\min(q_{\mathrm{MO}}, D_{\mathrm{MO}}) + (\alpha p_{\mathrm{IS}} + k)\min(q_{\mathrm{IS}}, D_{\mathrm{IS}}) - c_{\mathrm{MO}} q_{\mathrm{MO}}$$

$$u_{\mathrm{IS}} = (1 - \alpha) p_{\mathrm{IS}} \min(q_{\mathrm{IS}}, D_{\mathrm{IS}}) - c_{\mathrm{IS}} q_{\mathrm{IS}}$$

Demand functions

The seller j who sets the lower price faces the "original" demand function $Q(p_j)$

The seller i who sets the higher price faces the "residual" demand function $R(p_i; q_j, p_j)$

$D_i(p_i; q_j, p_j)$ = player i's demand when they set price p_i and the other player sets price p_j and quantity q_j

$$D_{\mathrm{IS}}(p_{\mathrm{IS}}; q_{\mathrm{MO}}, p_{\mathrm{MO}}) = \begin{cases} Q(p_{\mathrm{IS}}) & \text{if } p_{\mathrm{IS}} \leq p_{\mathrm{MO}} \\ R(p_{\mathrm{IS}}; q_{\mathrm{MO}}, p_{\mathrm{MO}}) & \text{if } p_{\mathrm{IS}} > p_{\mathrm{MO}} \end{cases}$$

$$D_{\mathrm{MO}}(p_{\mathrm{MO}}; q_{\mathrm{IS}}, p_{\mathrm{IS}}) = \begin{cases} R(p_{\mathrm{MO}}; q_{\mathrm{IS}}, p_{\mathrm{IS}}) & \text{if } p_{\mathrm{IS}} \leq p_{\mathrm{MO}} \\ Q(p_{\mathrm{MO}}) & \text{if } p_{\mathrm{IS}} > p_{\mathrm{MO}} \end{cases}$$

Demand functions (cont.)

We assume the "original" demand function is linear with unit slope.

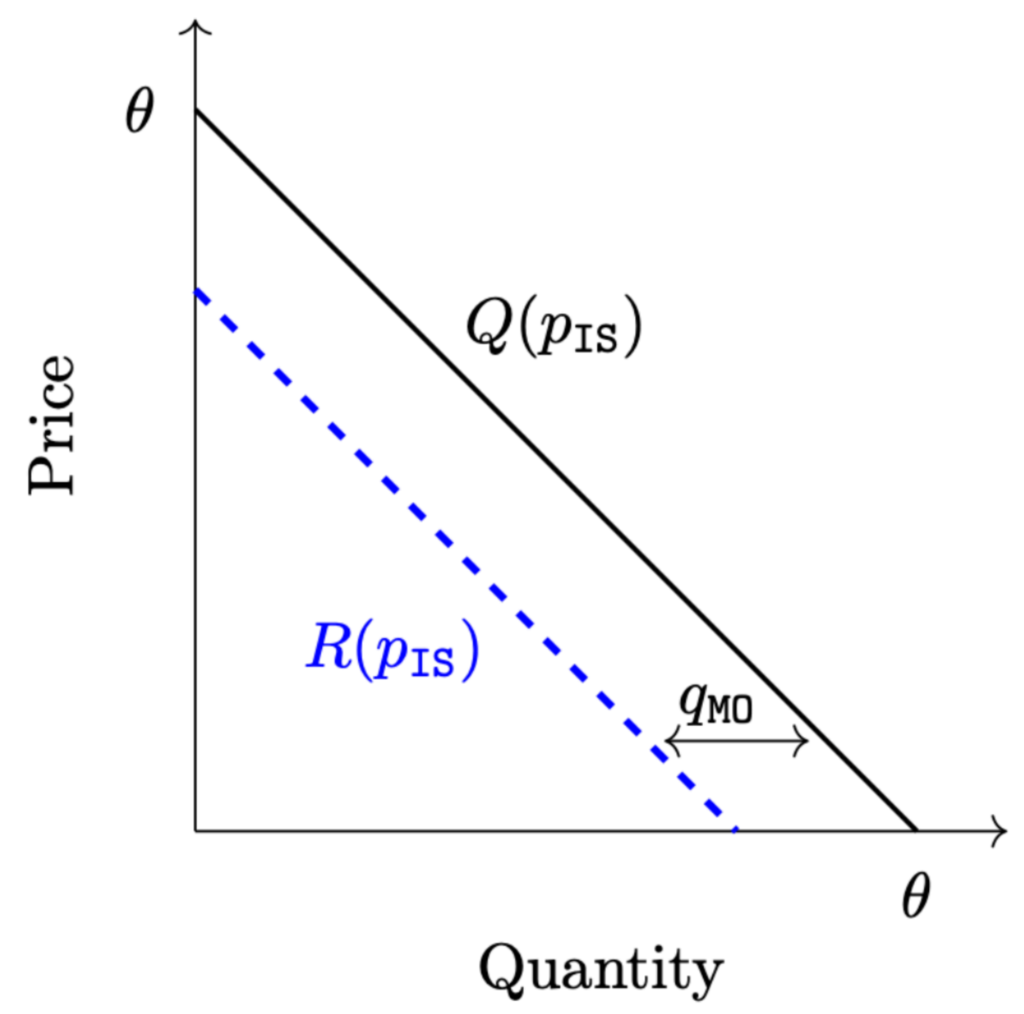
Assumption: The quantity demanded at price p is

$$Q(p) = \begin{cases} \theta - p & \text{for } 0 \le p \le \theta \\ 0 & \text{for } p > \theta \end{cases}$$

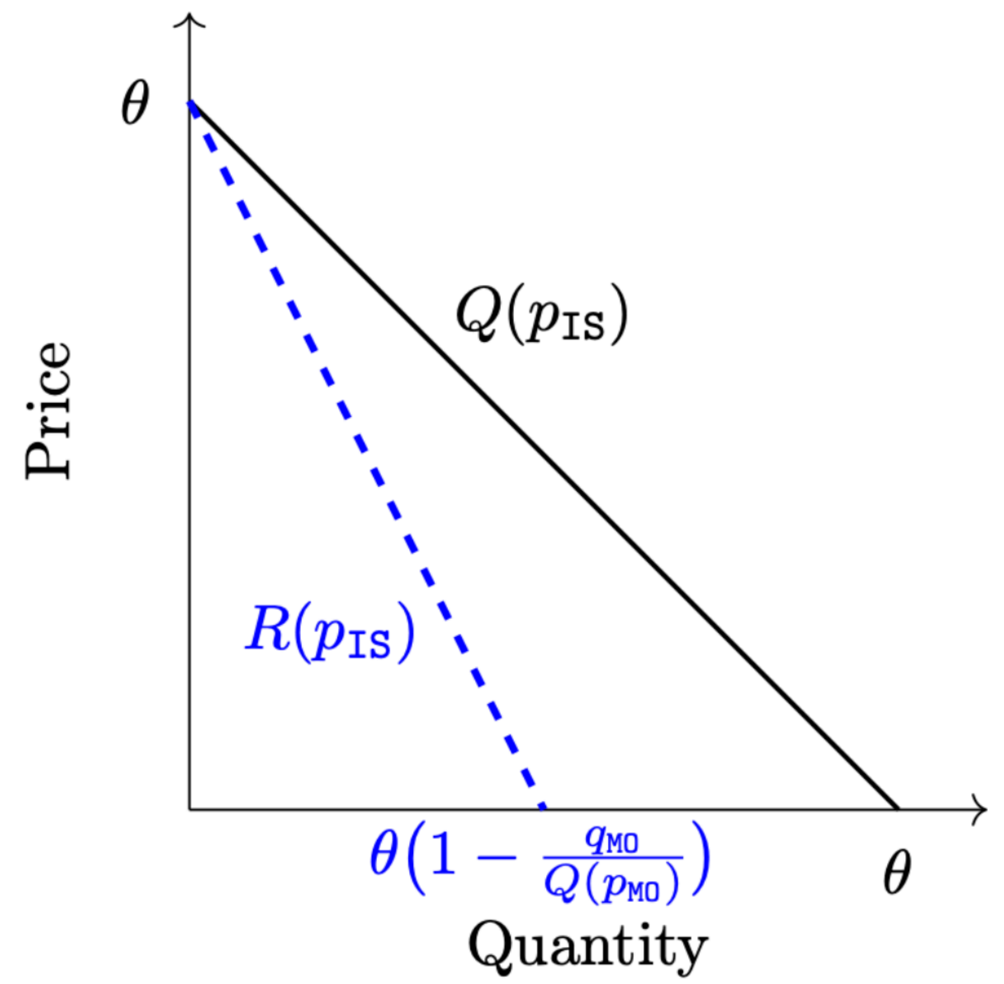
Demand functions (cont.)

The "residual" demand function $R(p_i; q_j, p_j)$ depends on the assumed **rationing rule**

Intensity rationing: customers with the highest valuation for the good arrive first (and buy at the lower price)



Proportional rationing: the probability a customer is able to buy at the lower price is independent of their valuation



Some key prices

IS's break-even price

$$p_0 = \frac{c_{\mathrm{IS}}}{1 - \alpha}$$

The price at which IS gets zero utility from selling the good

We assume that given the choice between selling a positive quantity at p_0 vs. not selling at all, they choose to sell at p_0

IS's optimal sole-seller price

$$p^{\star}_{\mathrm{IS}} = \frac{1}{2}(p_0 + \theta)$$

The price IS would set if MO were not a seller

Computing the Equilibrium

Solution concept

Let $a = (p_{\text{MO}}, q_{\text{MO}})$ be MO's action and let $b = (p_{\text{IS}}, q_{\text{IS}})$ be IS's action.

<u>Definition</u>: A strategy profile $(a^*, b^*(a))$ constitutes a ***subgame-perfect Nash equilibrium*** if and only if for every MO action a, IS's strategy $b^*(a)$ satisfies

$$b^*(a) = \arg\max_b u_{\text{IS}}(a, b)$$

and

$$a^* = \arg\max_a u_{\text{MO}}(a, b^*(a))$$

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$$\boxed{b^*(a) = \arg\max_b u_{\mathrm{IS}}(a, b)}$$

We will derive this first

and

$$a^* = \arg\max_a u_{\mathrm{MO}}(a, b^*(a))$$

Proposition:

(very informally) IS should always meet their demand

(less informally) Fixing p_{IS}, q_{MO}, & p_{MO}, it is utility-maximizing for IS to set $q_{\mathrm{IS}} = D_{\mathrm{IS}}(p_{\mathrm{IS}}; q_{\mathrm{MO}}, p_{\mathrm{MO}})$

$\implies$ *we can focus on deriving IS's best response just in terms of their price* p_{IS}

IS's possible strategies

1. **COMPETE** by setting $p_{\mathrm{IS}} \leq p_{\mathrm{MO}}$ (and thus face the original demand function)

2. **WAIT (IT OUT)** by setting $p_{\mathrm{IS}} > p_{\mathrm{MO}}$ (and thus face the residual demand function)

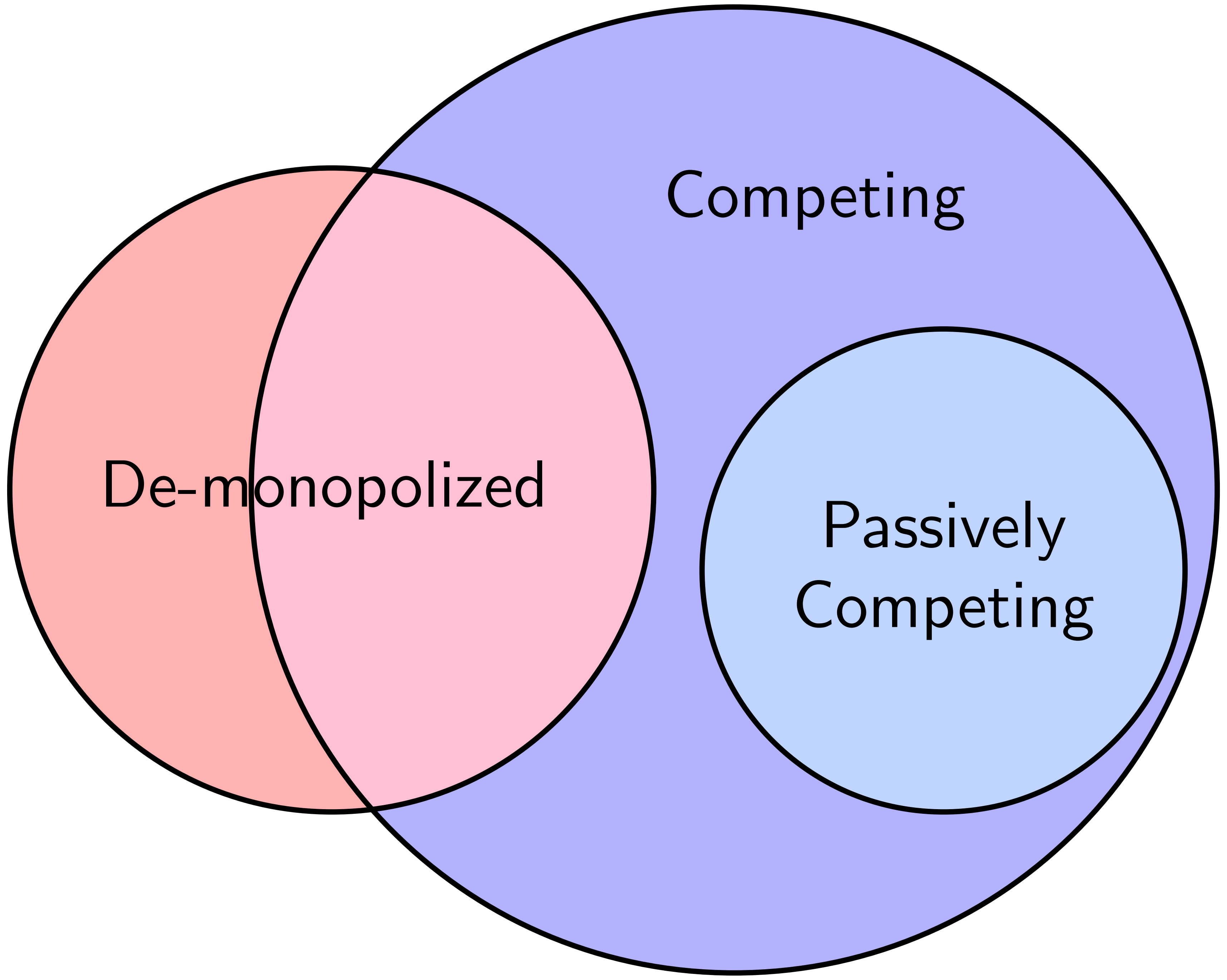
3. **ABSTAIN** by setting $p_{\mathrm{IS}} = \infty$

Describing competition

Definition: We say that IS has been de-monopolized if they set $p_{\mathrm{IS}} < p^{\star}_{\mathrm{IS}}$ (i.e., they set a price strictly lower than their optimal sole-seller price)

Definition: We say that IS is competing if they set $p_{\mathrm{IS}} \leq p_{\mathrm{MO}}$

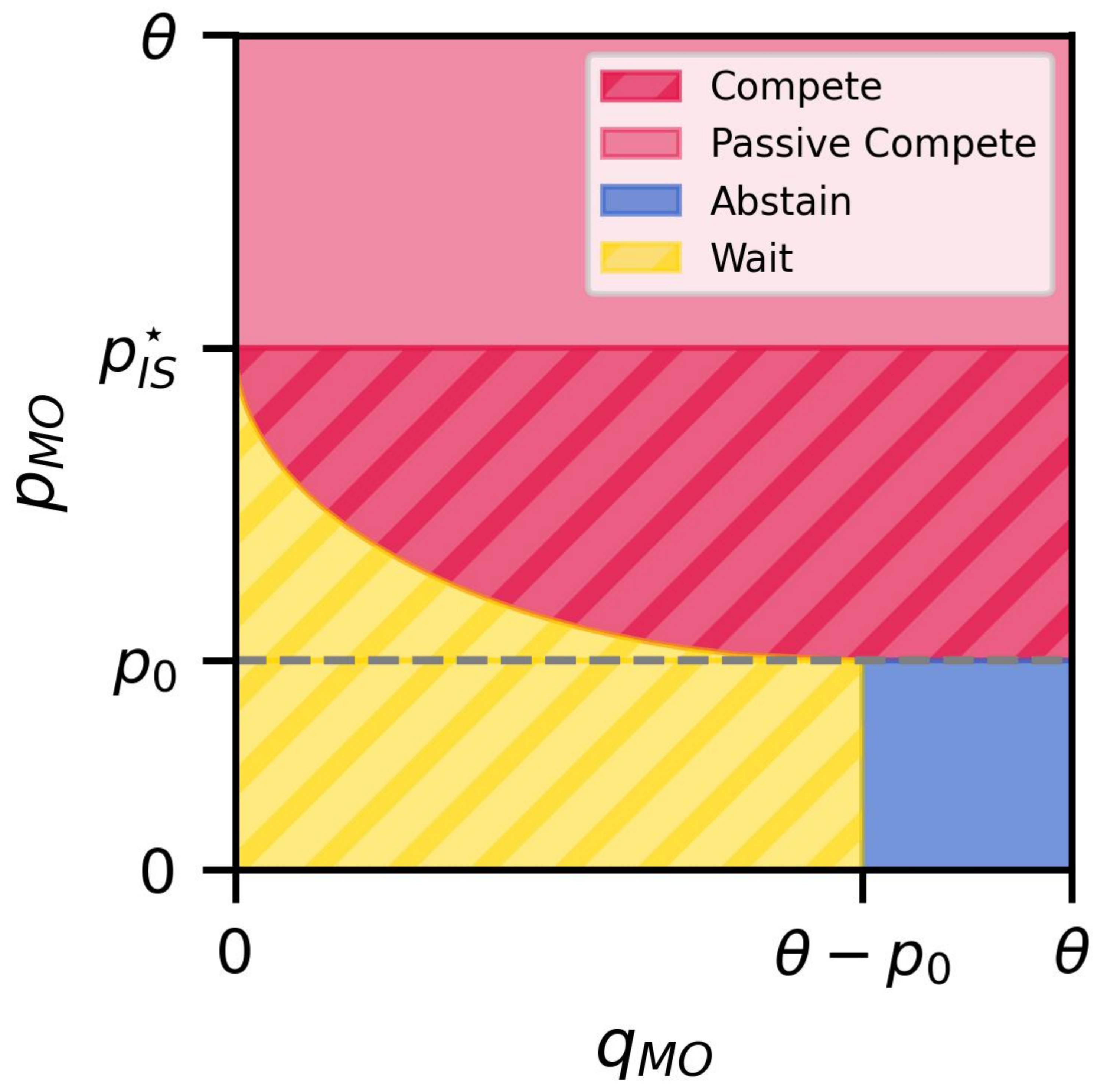
Definition: When MO's price is higher than IS's optimal sole-seller price ($p_{\mathrm{MO}} \geq p^{\star}_{\mathrm{IS}}$), we say that IS is passively competing if they set $p_{\mathrm{IS}} = p^{\star}_{\mathrm{IS}} \leq p_{\mathrm{MO}}$



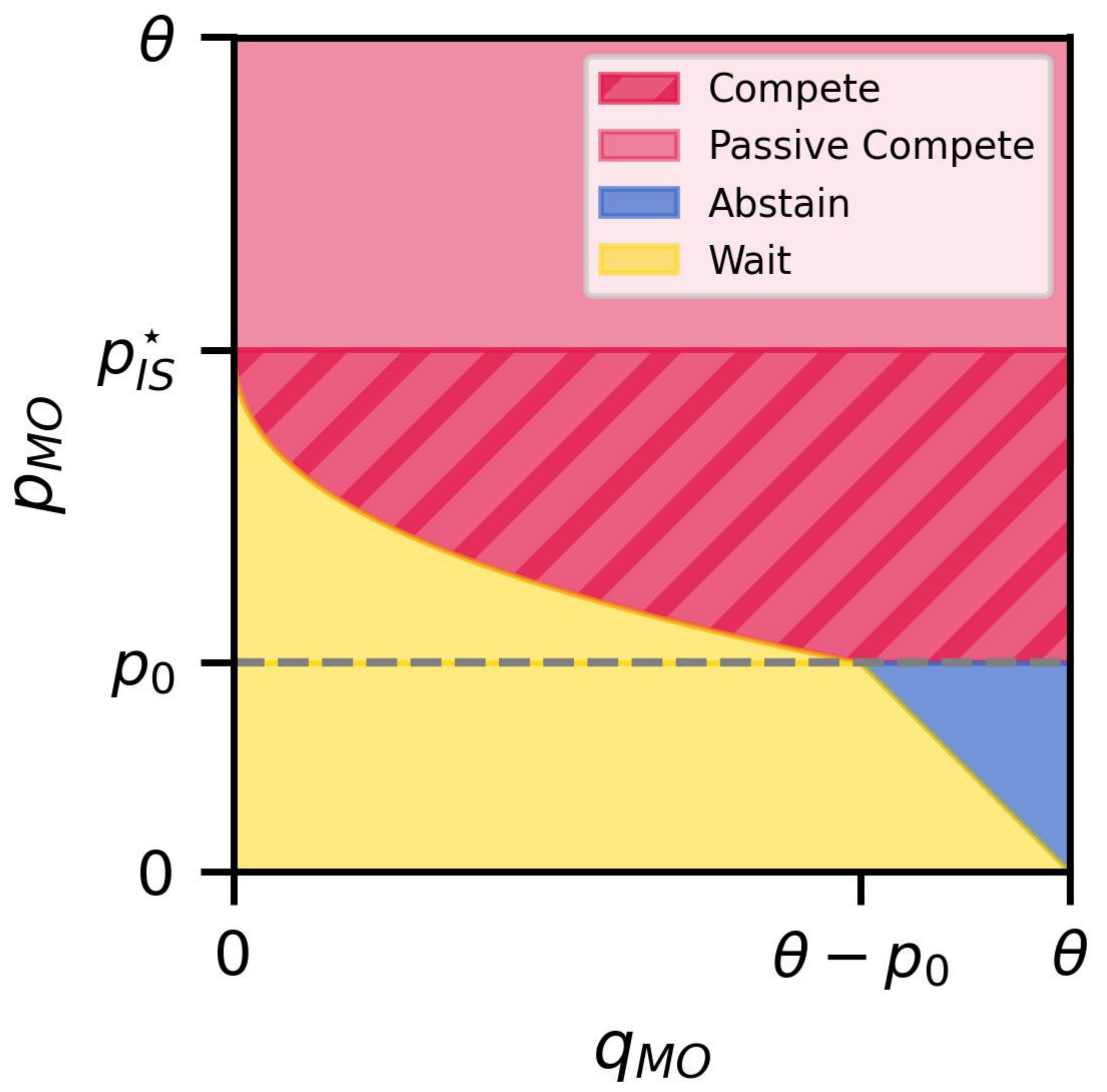
p_0 = IS's break-even price $\quad$ $p^{\star}_{\mathrm{IS}}$ = IS's optimal sole-seller price

What is IS's optimal strategy given MO's choice of $(p_{\mathrm{MO}}, q_{\mathrm{MO}})$?

Intensity rationing



Proportional rationing



Stripes denote de-monopolization

p_0 = IS's break-even price $p^{\star}_{\mathrm{IS}}$ = IS's optimal sole-seller price

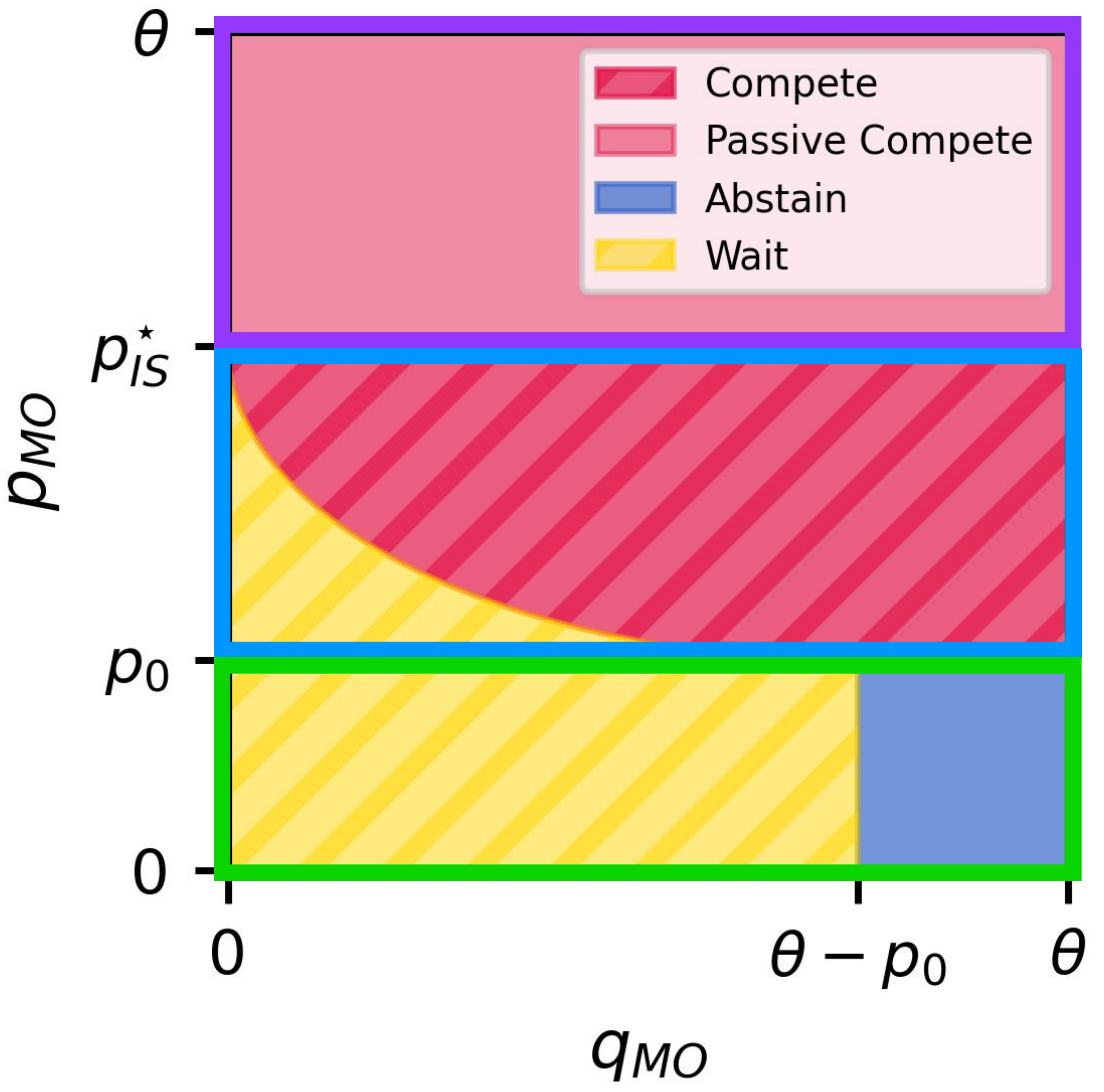
How did we get this?

Proof intuition: When MO sets

.... a high price, IS can set their optimal sole-seller price and still get all the demand ("passively competes").

... an intermediate price, IS must decide between (1) competing, by matching MO's price, or (2) waiting for MO to run out of inventory then setting whatever price they want.

... a low price, IS cannot get positive utility by matching MO's price, so they will wait for MO to sell out. If there is no demand left at price p_0 or higher after MO sells out, IS will abstain.



p_0 = IS's break-even price $p^\star_{\mathrm{IS}}$ = IS's optimal sole-seller price

IS's best response, in full detail

Proposition (large $p_{p_{\mathrm{MO}}}$): Whenever $p_{\mathrm{MO}} \geq p^{\star}_{\mathrm{IS}}$, IS passively competes by setting $p_{\mathrm{IS}} = p^{\star}_{\mathrm{IS}}$.

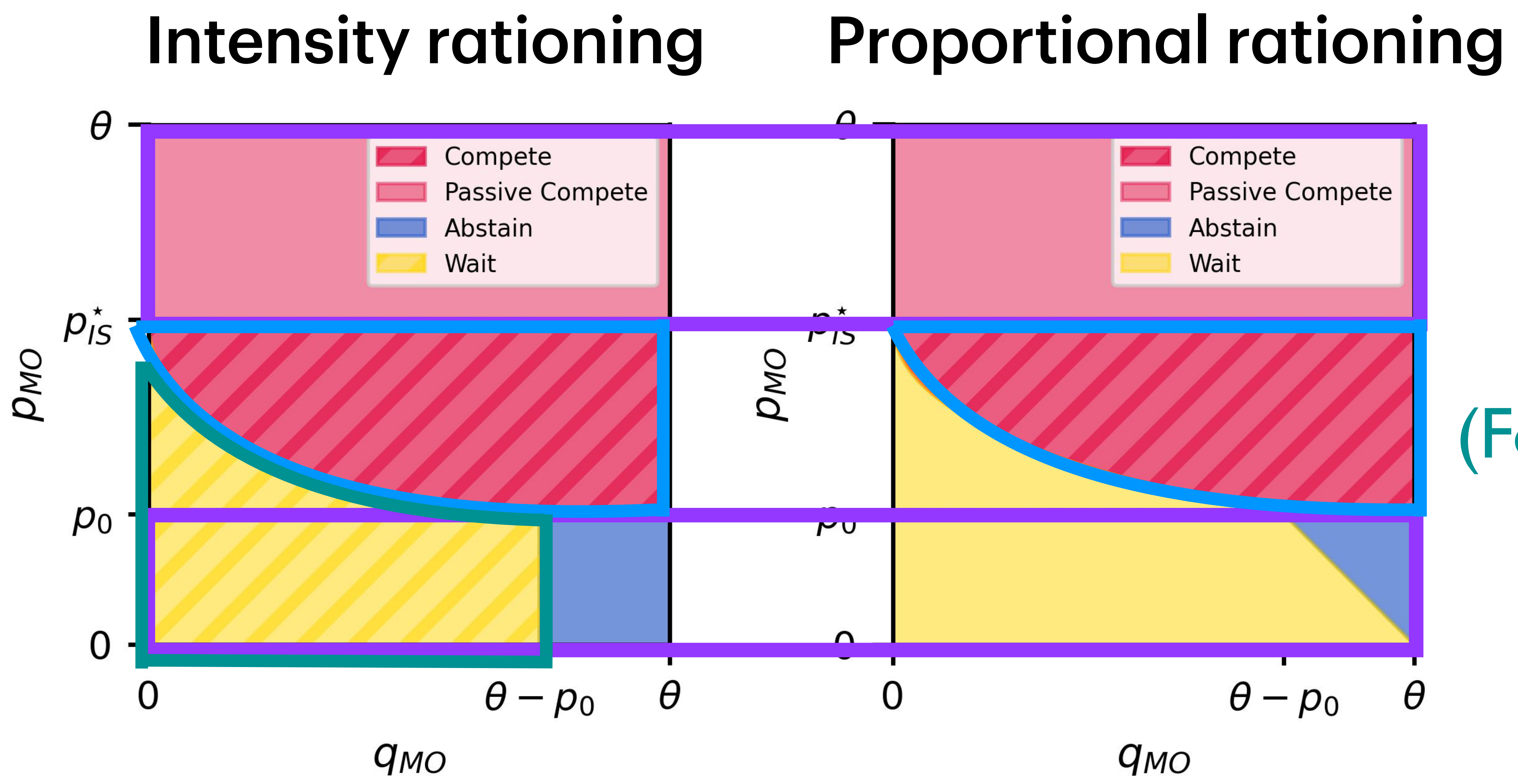
Proposition (intermediate $p_{p_{\mathrm{MO}}}$): Let $p_{\mathrm{MO}} \in [p_0, p^{\star}_{\mathrm{IS}})$. IS's response depends on MO's inventory, relative to a threshold $q^{\dagger}(p_{\mathrm{MO}})$: if $q_{\mathrm{MO}} \geq q^{\dagger}(p_{\mathrm{MO}})$, IS competes by setting $p_{\mathrm{IS}} = p_{\mathrm{MO}}$; otherwise they wait it out by setting $p_{\mathrm{IS}} = p^{W}_{\mathrm{IS}}$.

Proposition (small $p_{p_{\mathrm{MO}}}$): Let $p_{\mathrm{MO}} < p_0$. IS's response depends on MO's inventory, relative to a threshold $q^{\ddagger}(p_{\mathrm{MO}})$: if $q_{\mathrm{MO}} \geq q^{\ddagger}(p_{\mathrm{MO}})$, IS abstains; otherwise they wait it out by setting $p_{\mathrm{IS}} = p^{W}_{\mathrm{IS}}$.

		Intensity	Proportional
Compete threshold for $p_{\mathrm{MO}} \in [p_0, p^{\star}_{\mathrm{IS}})$	$\boxed{q^{\dagger}(p_{\mathrm{MO}})}$	$\theta - p_0 - 2\sqrt{(p_{\mathrm{MO}} - p_0)(\theta - p_{\mathrm{MO}})}$	$Q(p_{\mathrm{MO}})\left(1 - \frac{(p_{\mathrm{MO}} - p_0)(\theta - p_{\mathrm{MO}})}{(p^{\star}_{\mathrm{IS}} - p_0)(\theta - p^{\star}_{\mathrm{IS}})}\right)$
Abstain threshold for $p_{\mathrm{MO}} < p_0$	$\boxed{q^{\ddagger}(p_{\mathrm{MO}})}$	$\theta - p_0$	$Q(p_{\mathrm{MO}})$
IS price when waiting	$\boxed{p^{W}_{\mathrm{IS}}}$	$p^{\star}_{\mathrm{IS}} - \frac{q_{\mathrm{MO}}}{2}$	$p^{\star}_{\mathrm{IS}}$

p_0 = IS's break-even price $\quad$ $p^{\star}_{\mathrm{IS}}$ = IS's optimal sole-seller price

Implications for competition



Stripes denote de-monopolization

$p_{\mathrm{MO}} \geq p^\star_{\mathrm{IS}}$ → IS is not de-monopolized

$p_{\mathrm{MO}} < p^\star_{\mathrm{IS}}$ and IS competes → IS is de-monopolized

(For intensity rationing) IS waits → IS is de-monopolized

$p_{\mathrm{MO}} < p_0$ → IS does not compete

p_0 = IS's break-even price $\quad$ $p^\star_{\mathrm{IS}}$ = IS's optimal sole-seller price

Solution concept

Let $a = (p_{\text{MO}}, q_{\text{MO}})$ be MO's action and let $b = (p_{\text{IS}}, q_{\text{IS}})$ be IS's action.

<u>Definition</u>: A strategy profile $(a^*, b^*(a))$ constitutes a ***subgame-perfect Nash equilibrium*** if and only if for every MO action a, IS's strategy $b^*(a)$ satisfies

$$b^*(a) = \arg\max_b u_{\text{IS}}(a, b)$$

We just derived this (IS's best response function)

and

$$a^* = \arg\max_a u_{\text{MO}}(a, b^*(a))$$

Now we have to plug into this

MO’s possible strategies

1. **INDUCE ABSTAIN** by setting a price low enough and quantity high enough that IS’s best response is to abstain.
2. **INDUCE COMPETE** by setting a moderate price and quantity high enough that IS’s best response is to compete.
3. **INDUCE WAIT** by setting a price low enough and quantity low enough that IS’s best response is to wait.
   - Note: we include $q_{\text{MO}} = 0$ (MO abstains) in this bucket.

Equilibrium in constrained game

Before solving for the equilibrium in the full game, we will consider a *constrained* game

Full Game

1. MO chooses $p_{\mathrm{MO}}, q_{\mathrm{MO}}$
2. IS chooses $p_{\mathrm{IS}}, q_{\mathrm{IS}}$

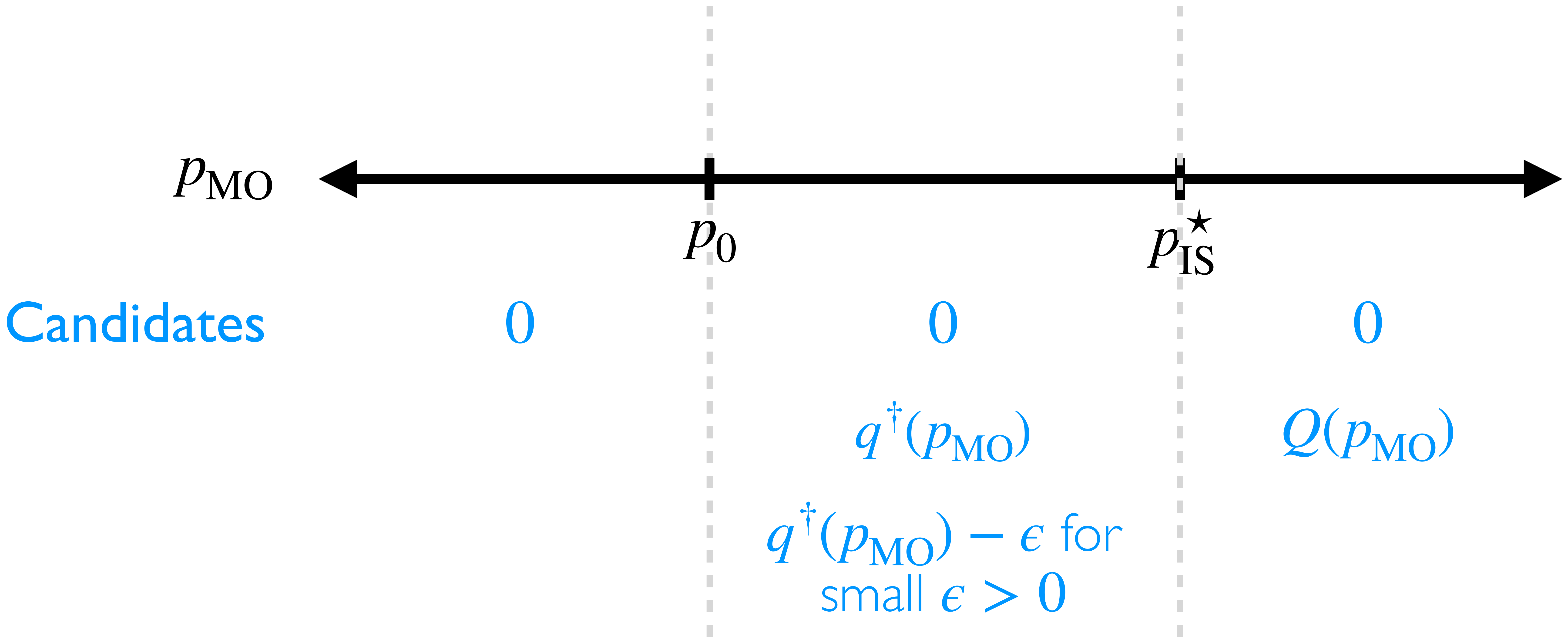
Constrained Game

p_{MO} is fixed.

1. MO chooses q_{MO}
2. IS chooses $p_{\mathrm{IS}}, q_{\mathrm{IS}}$

Equilibrium in constrained game (intuition)

Depending on the value of p_{MO}, there are only a couple of candidate q_{MO} values that could be optimal (via straightforward 2nd derivative arguments).



p_0 = IS's break-even price $p_{\mathrm{IS}}^\star$ = IS's optimal sole-seller price $q^\dagger(p_{\mathrm{MO}})$ = smallest MO inventory at which IS competes, when MO's price is p_{MO}

Equilibrium in constrained game (in full detail)

Lemma: Given a fixed p_{MO}, the following is an optimal MO inventory

$$q^*_{\text{MO}}(p_{\text{MO}}) = \begin{cases} 0, & \text{if } p_{\text{MO}} \geq p^\star_{\text{IS}} \\ \arg\max_{q \in \{0, q^\dagger(p_{\text{MO}}), q^\dagger(p_{\text{MO}}) - \epsilon\}} u_{\text{MO}}(p_{\text{MO}}, q), & \text{if } p_{\text{MO}} \in [p_0, p^\star_{\text{IS}}) \\ \arg\max_{q \in \{0, Q(p_{\text{MO}})\}} u_{\text{MO}}(p_{\text{MO}}, q), & \text{if } p_{\text{MO}} < p_0 \,. \end{cases}$$

The optimal quantity is unique except for when there are ties in the argmax.

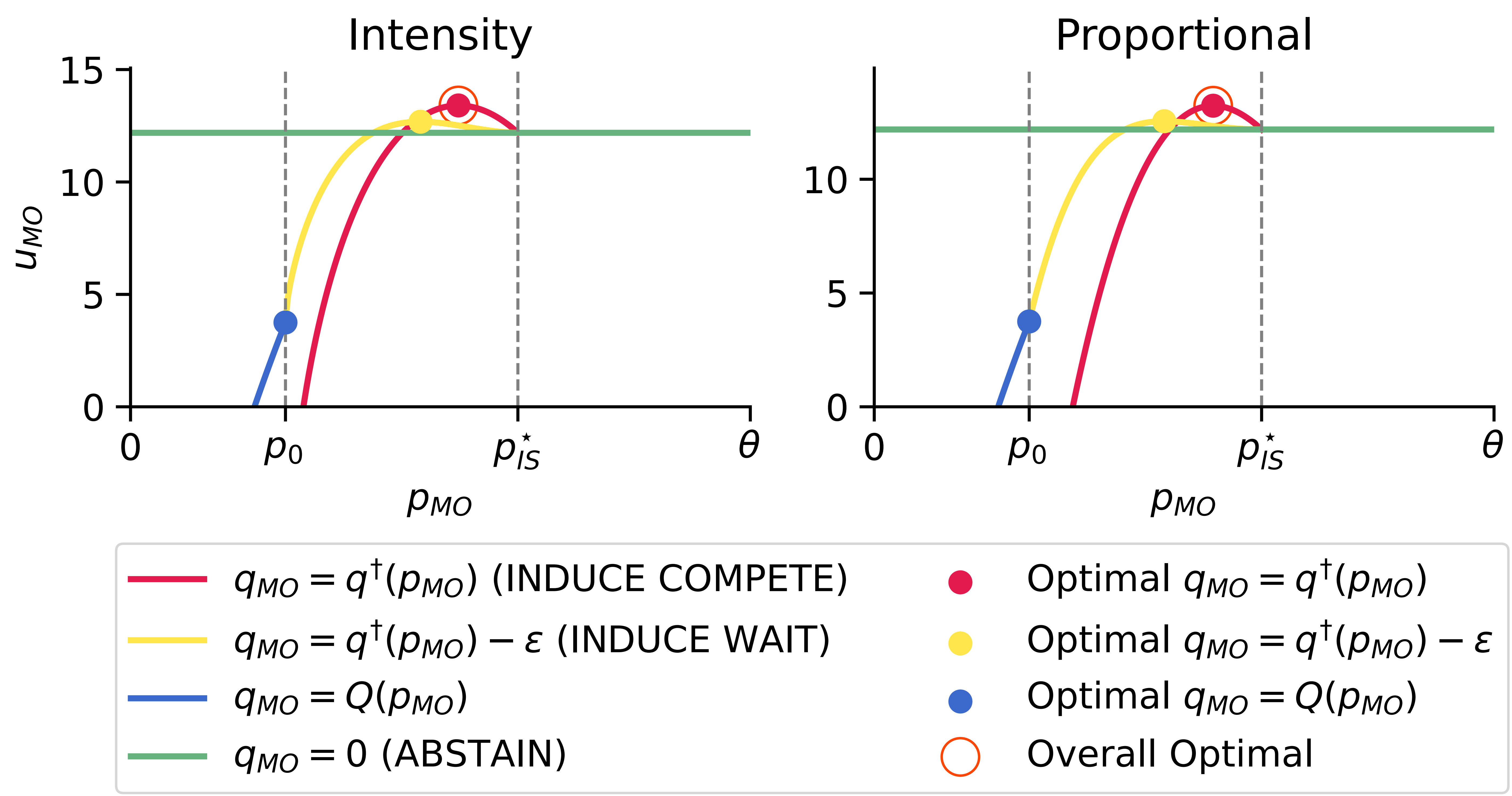
$u_{\text{MO}}(p, q)$ = MO's utility when they set price p and quantity q and IS plays their best response

p_0 = IS's break-even price $p^\star_{\text{IS}}$ = IS's optimal sole-seller price $q^\dagger(p_{\text{MO}})$ = smallest MO inventory at which IS competes, when MO's price is p_{MO}

Equilibrium of full game

To get the equilibrium of the full game where MO can choose their quantity ***and price***, we can plug the candidate optimal quantities into u_{MO} and optimize over p_{MO}

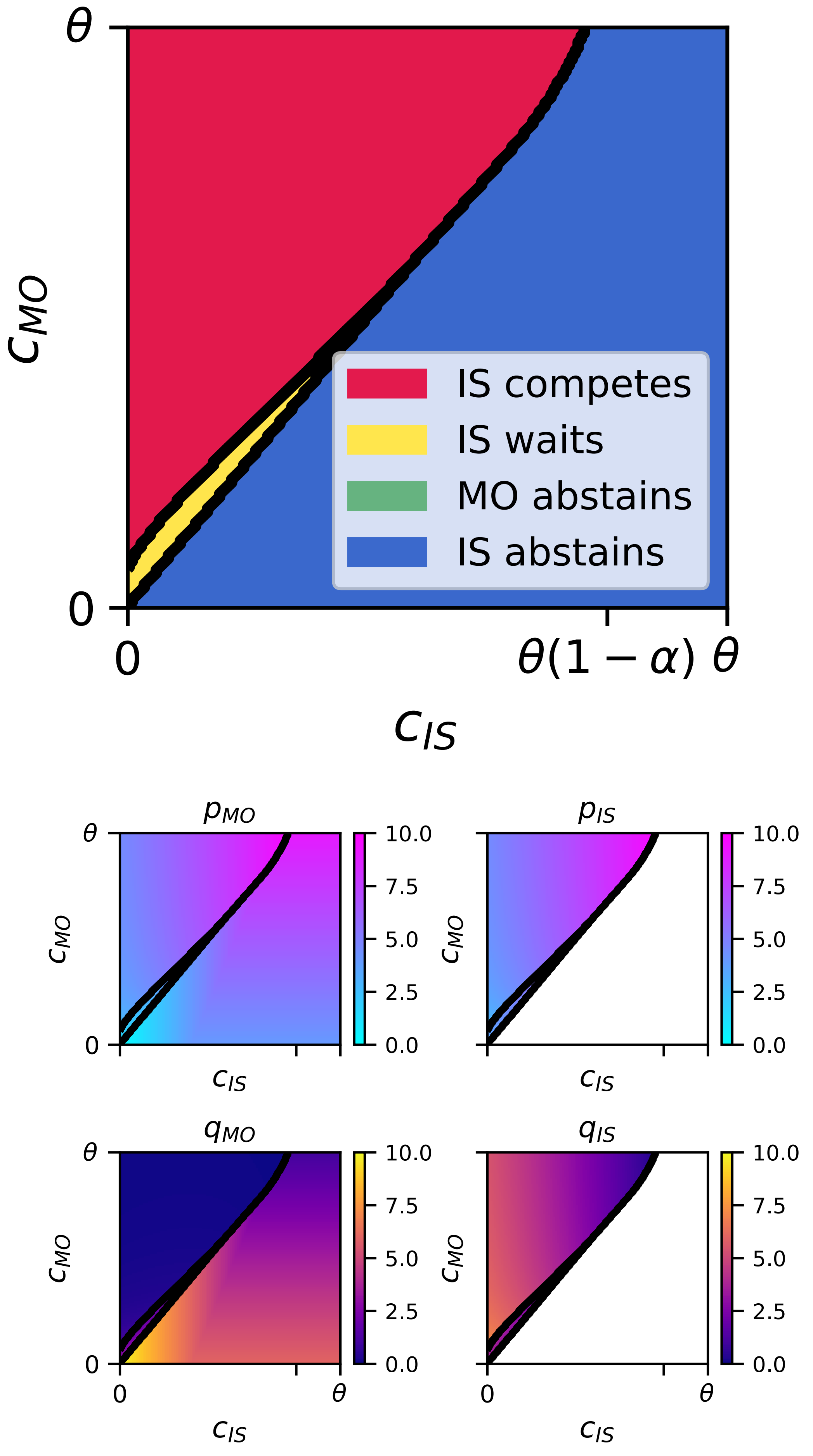
i.e., to identify the equilibrium, we just have to solve three single-variable optimization problems and compare the resulting utilities



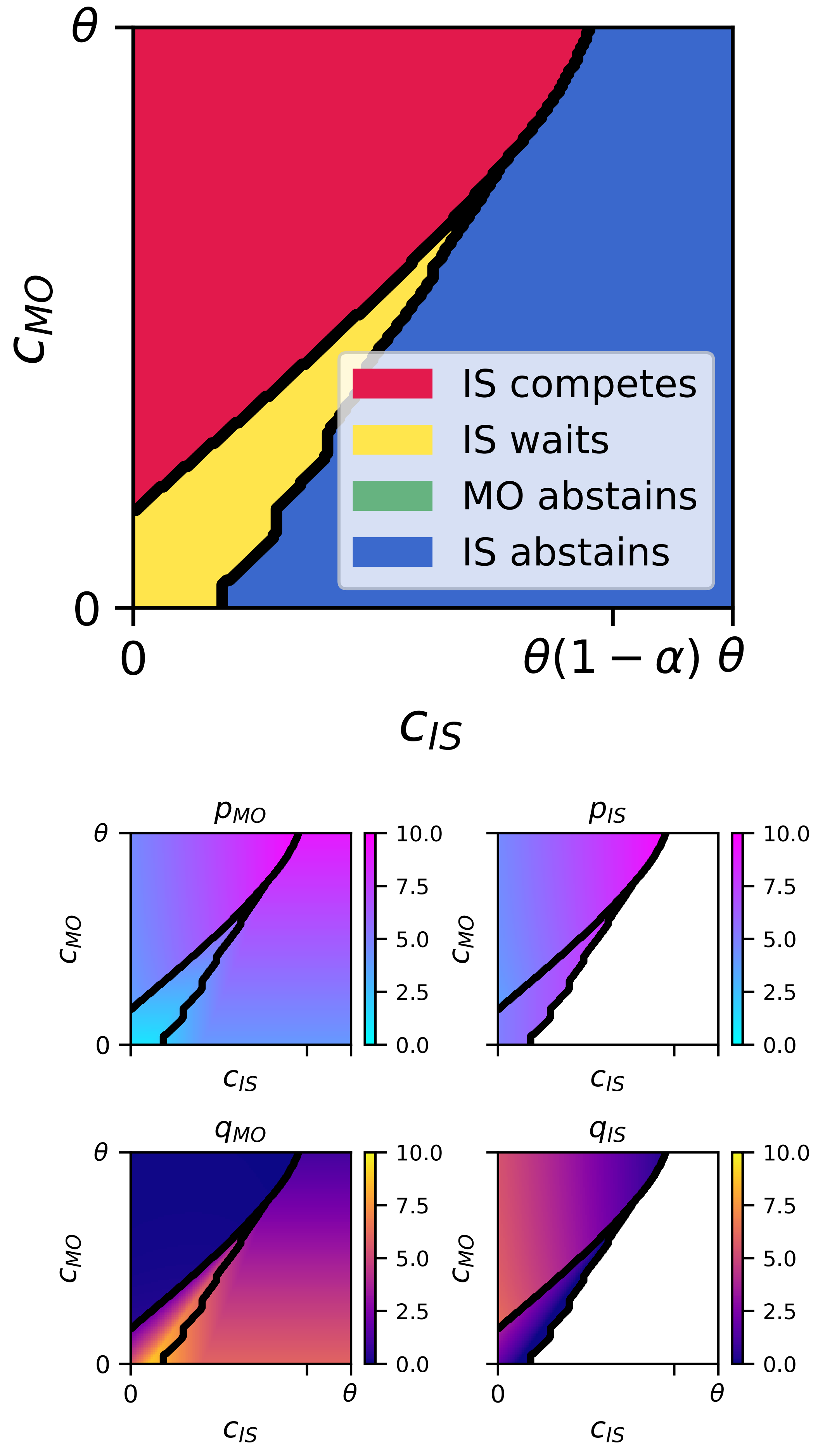
Implications of the Equilibrium

How does the equilibrium change depending on the relative costs of the two sellers?

Intensity rationing



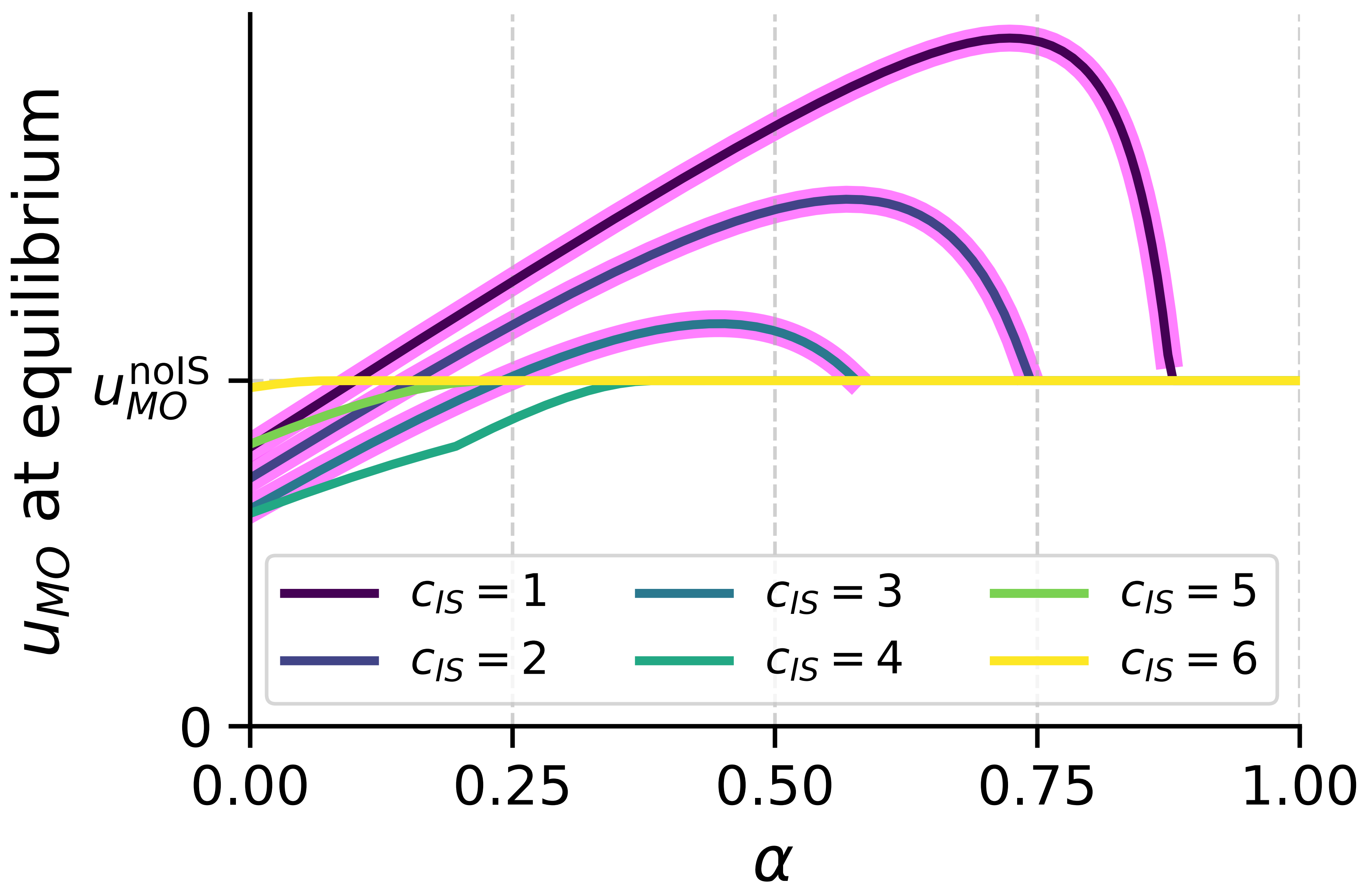
Proportional rationing



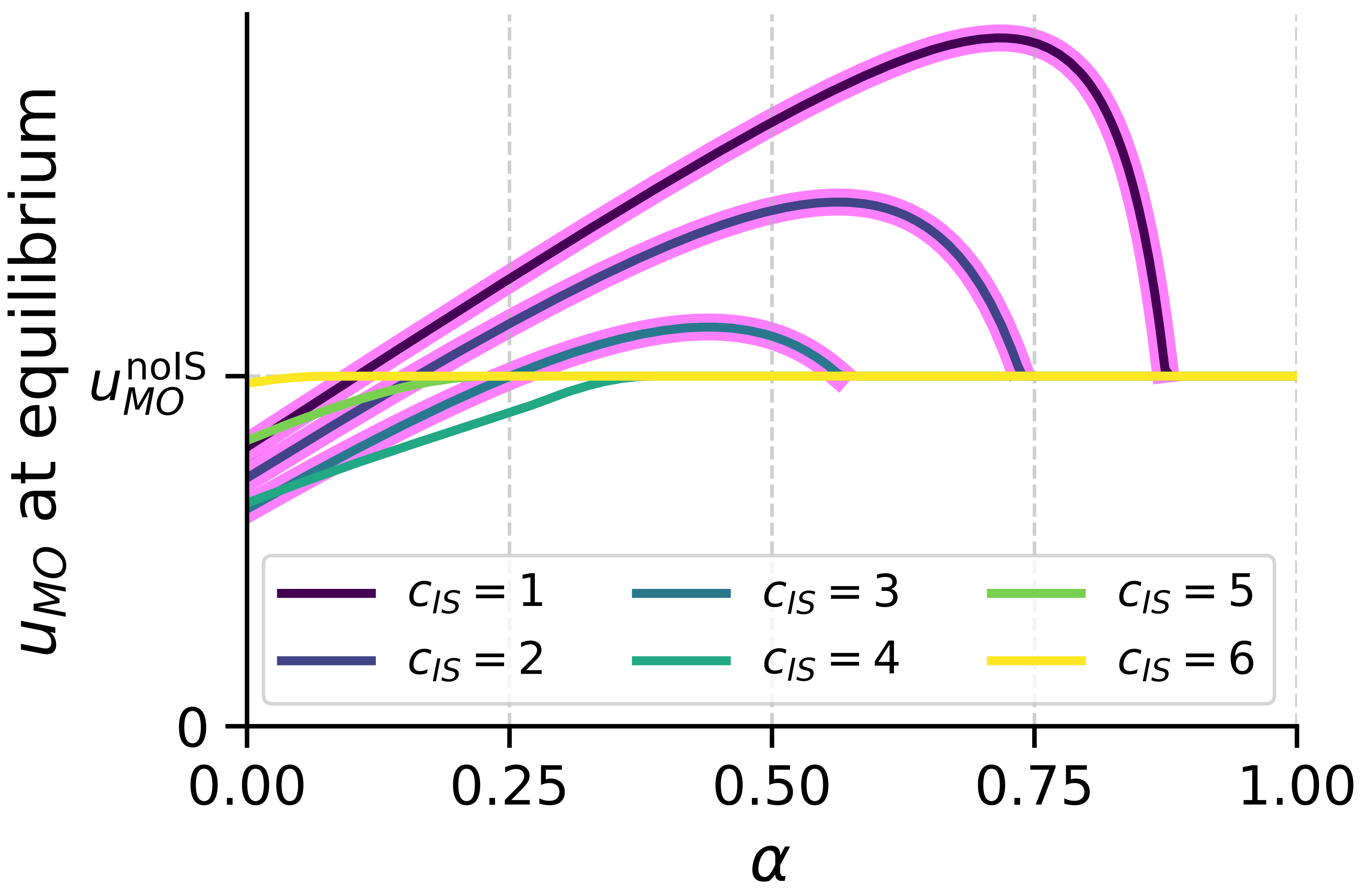
How should the marketplace operator set the referral fee α?

Recall: α = fraction of revenue IS must pay to MO

Intensity rationing



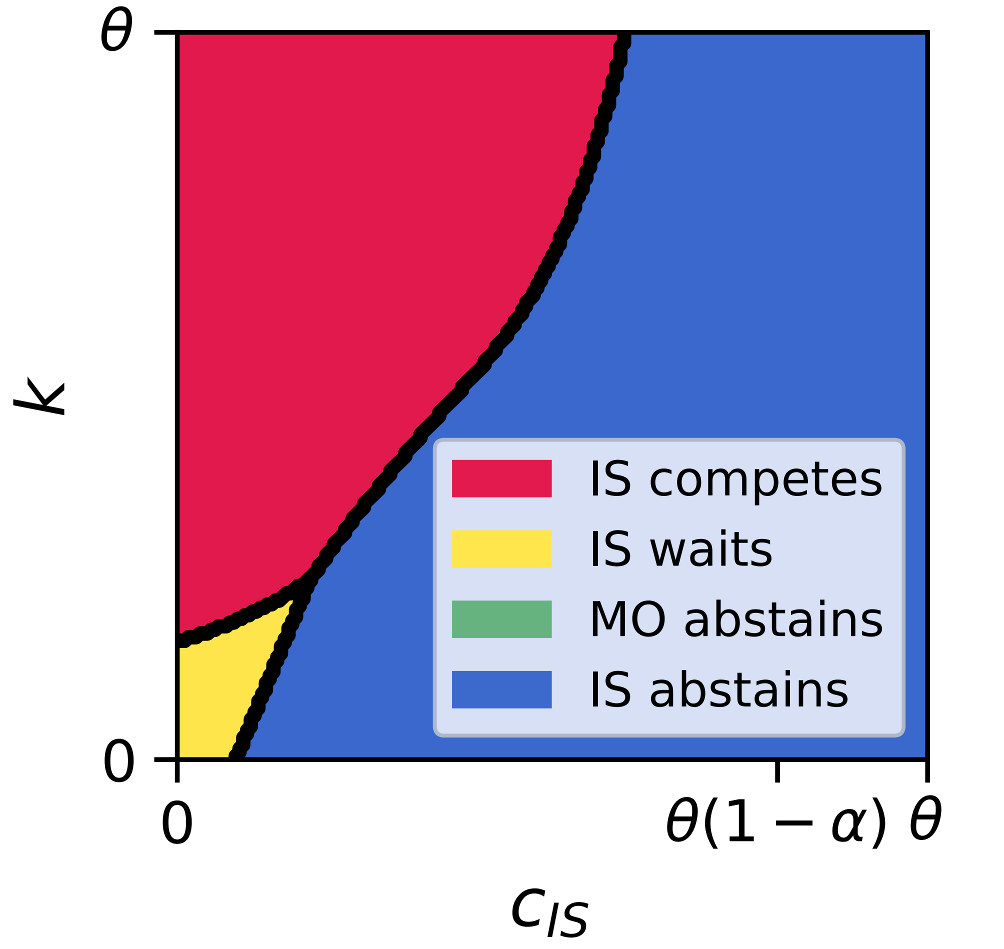
Proportional rationing

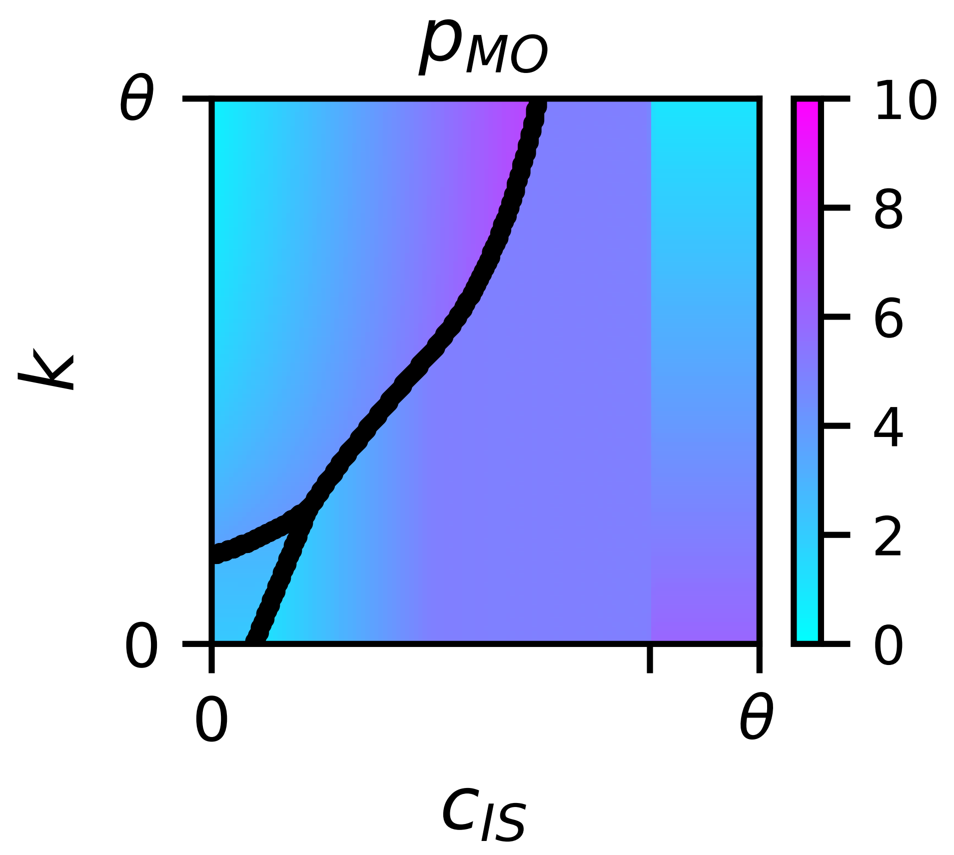


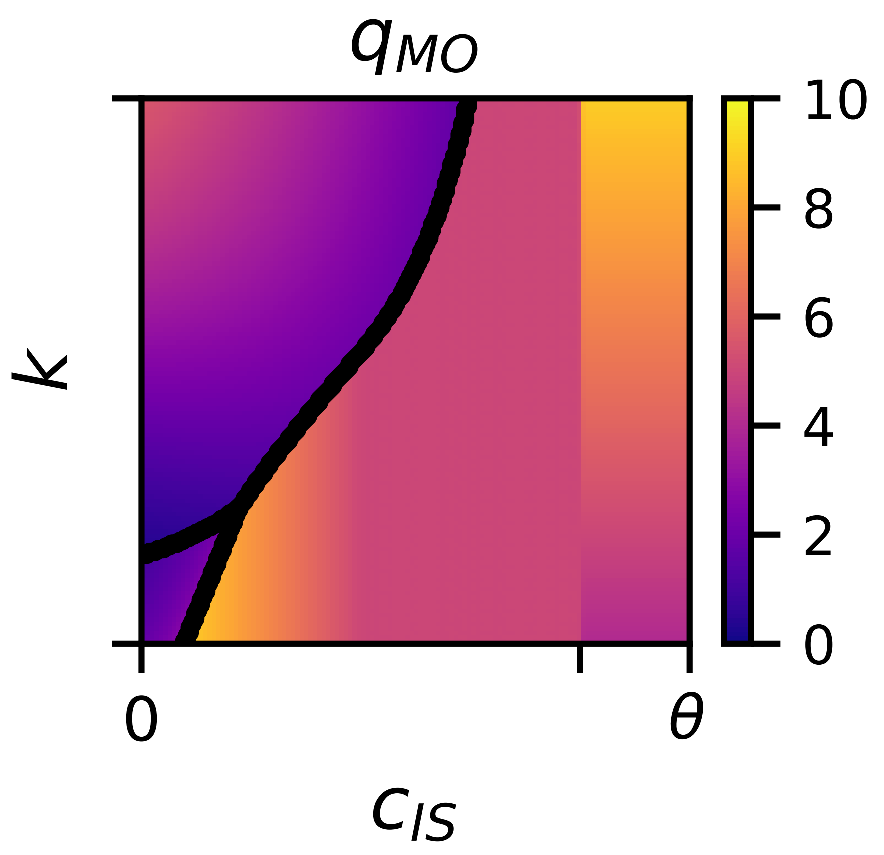
Pink highlighting means that IS competes at the equilibrium induced by those game parameters

Should MO behave differently when selling a product with a larger impact on customer experience (large k)?

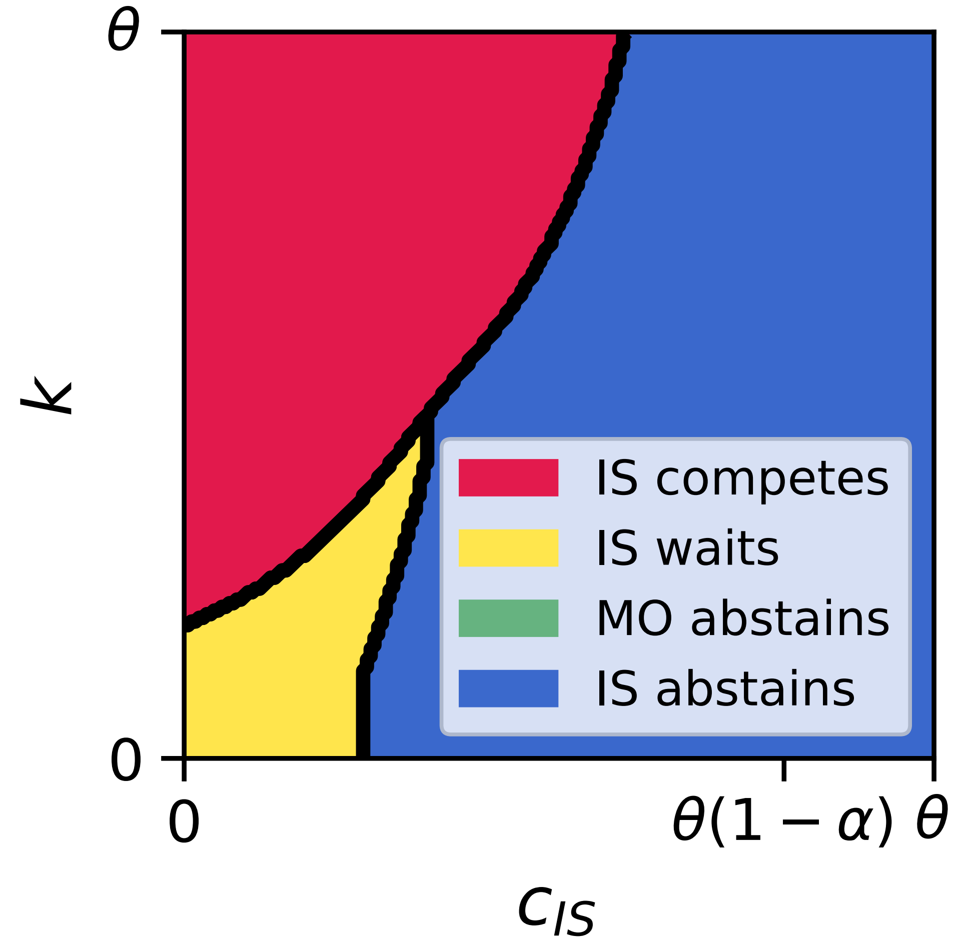
Intensity rationing

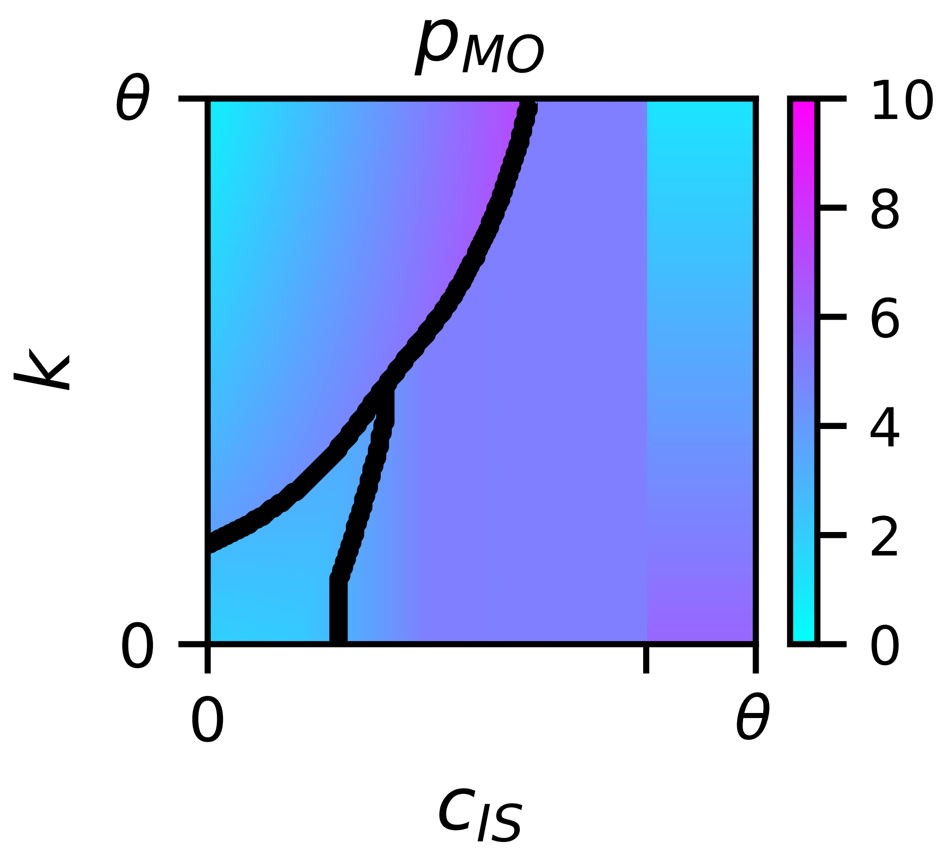


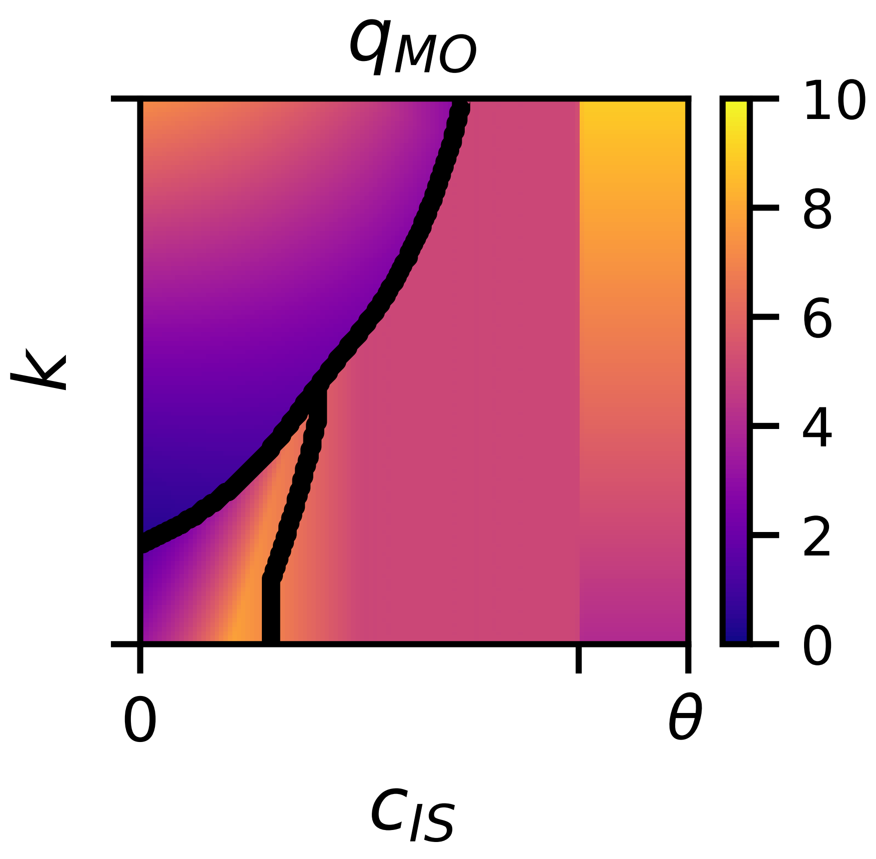




Proportional rationing







How does MO entering the market affect consumer surplus?

Lemma: Under intensity rationing with perfect substitutes, for any p_{MO} and q_{MO} (including the equilibrium values), as long as the independent seller best responds, the consumer surplus will be at least as high as if MO did not participate as a seller.

When $p_{\mathrm{MO}} \leq p_{\mathrm{IS}}^{\star}$, *MO's entry strictly increases consumer surplus.*

How does MO entering the market affect total welfare?

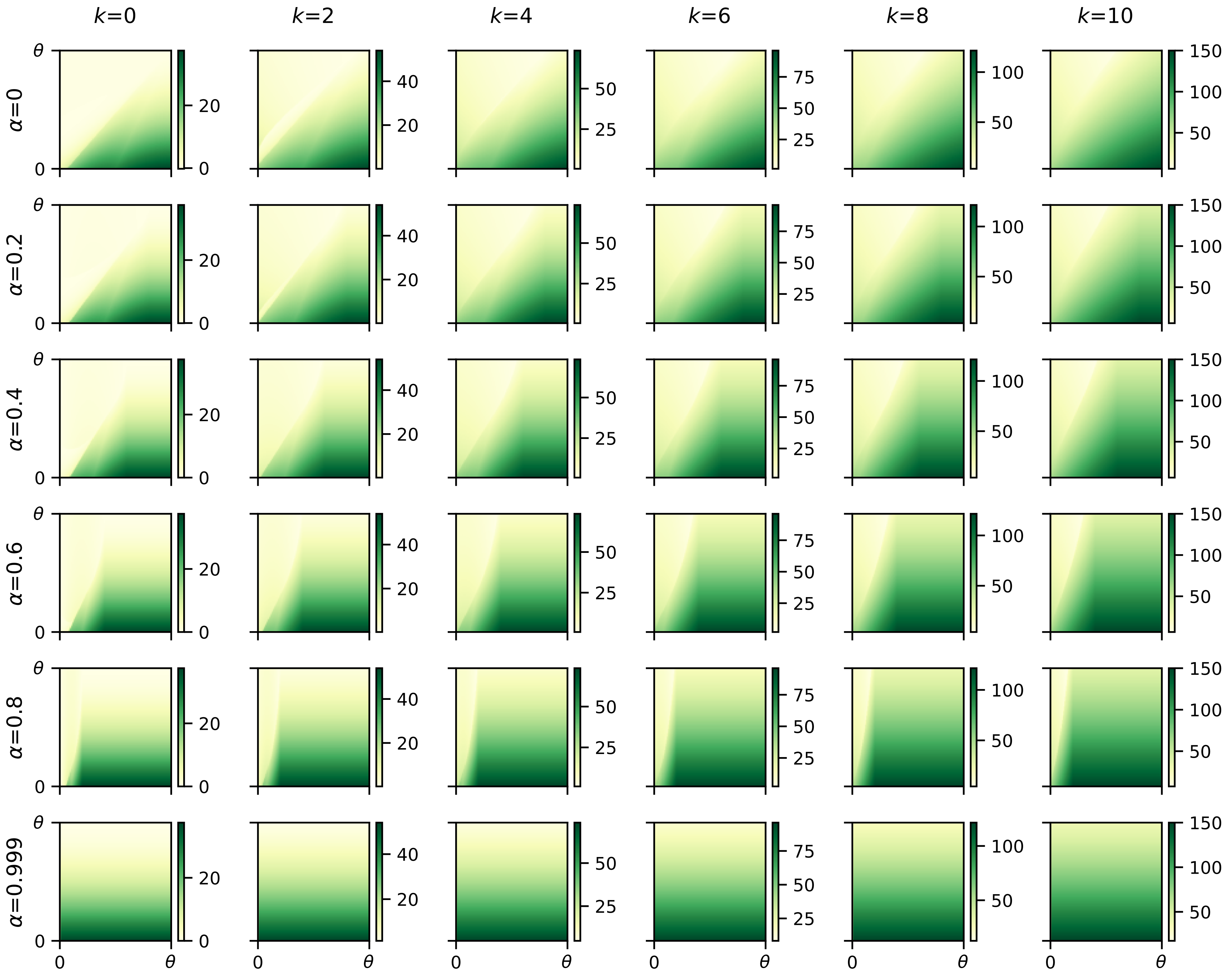
(Under intensity rationing)

Total welfare = consumer surplus + MO's utility + IS's utility

Computing

(welfare with MO & IS) - (welfare with only IS)

for many combinations of game parameters reveals that this difference is always **non-negative**



x-axis and y-axis of each individual plot are c_{IS} and c_{MO}, respectively

Conclusion

Summary and future directions

Summary: In online marketplaces, we have a duopoly in which one player is both the marketplace operator *and* a seller.

- We formulate this as a game and solve for the equilibrium
- Our analysis can be used to guide marketplace operators' policies

Directions for future work:

1. **Determining** welfare implications under rationing rules other than intensity rationing
2. **Robustness checks:** what happens under non-linear demand or with integer-constrained inventory?
3. **Improving model's realism:** replacing k with a multiplier of consumer surplus; introducing a positive salvage value
4. **Modeling extensions:** what happens with multiple independent sellers? what happens when timing is endogenous (i.e., MO can choose between the Stackelberg game and a simultaneous game)?

Thank you!

Paper available at arxiv.org/abs/2503.06582

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